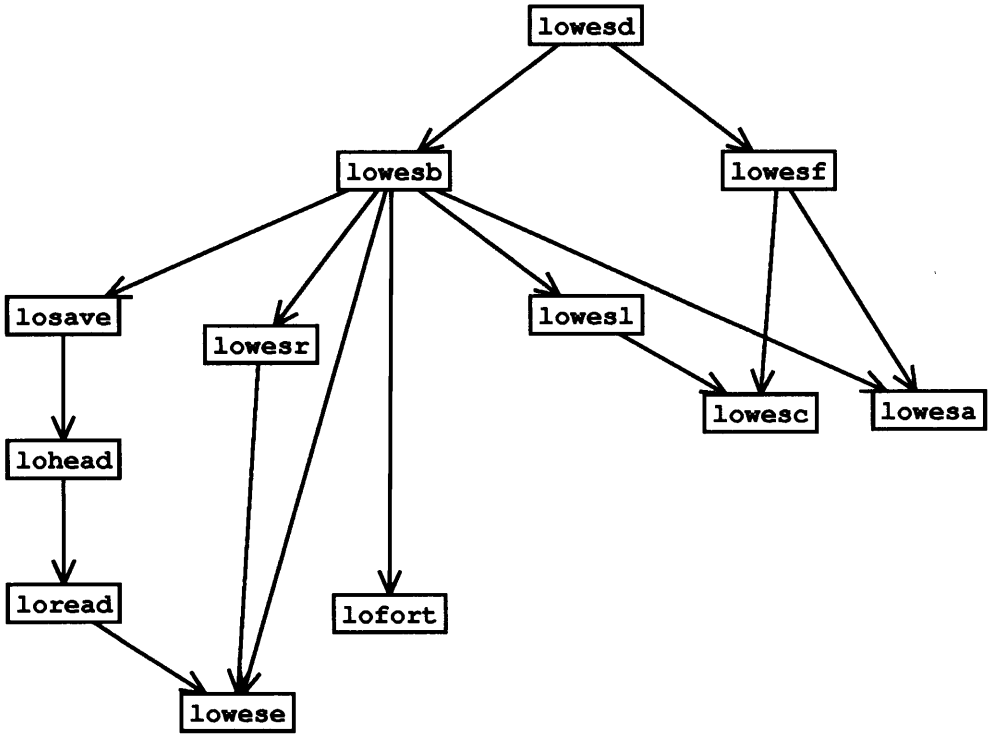
lowesd
lowesb
lowesf
losave
lowesr
lowesl
lowesc
lowesa
lohead
loread
lofort
lowese

kd

$dMAX \rightarrow 8$
$d2MAX \rightarrow 15$
$vcMAX \rightarrow 256$
Codemacros: ...eqn
$Lf \in \mathbf{R}^{0:d \times nvmax \times nf}$, *Old*, ... coefficients for blending
$Lq \in \mathbf{Z}^{nvmax \times nf}$, *Old*, ... indices of data points influencing each vertex
Typemacros: *dMAX*(*dMAX*,8), *d2MAX*(*d2MAX*,15), *vcMAX*(*vcMAX*,256), ...eqn
$a \in \mathbf{Z}^{n_{cell}}$, *Old*(*ncmax*), ... cut coordinate (0 for leaf)
$c \in \mathbf{Z}^{vc \times n_{cell}}$, *Old*(*vc*,*ncmax*), ... pointers to cell vertices
$cdeg \in \mathbf{Z}^{d}$, *Old*(*dMAX*)
$d \in \mathbf{Z}$, ... number of dimensions
$dd \in \mathbf{Z}$, ... only variables 1:dd enter distance calculation
$dist \in \mathbf{R}^{n}$, *Old*
$\eta \in \mathbf{R}^{nf}$, *Old*
$f_{cell} \in \mathbf{Z}$, ... cell count stopping tolerance
$f_{diam} \in \mathbf{R}$, ... cell diameter stopping tolerance
$hi \in \mathbf{Z}^{n_{cell}}$, *Old*(*ncmax*), ... right subcell or point
$kernel \in \mathbf{Z}$, ... which kind of weight function
$lo \in \mathbf{Z}^{n_{cell}}$, *Old*(*ncmax*), ... left subcell or point
$n \in \mathbf{Z}$, ... number of points
$n_{cell} \in \mathbf{Z}$, ... number of cells
$n_{vert} \in \mathbf{Z}$, ... number of vertices
$ncmax, nf \in \mathbf{Z}$
$nvmax \in \mathbf{Z}$, ... allocated size of v
$\pi \in \mathbf{Z}^{n}$, *Permutation*, *Old*, ... with *lo*, *hi* lists points in cells
plot: *Plot*
$\psi \in \mathbf{Z}^{n}$, *Permutation*, *Old*, ... workspace for sorting distances
$rw \in \mathbf{R}^{n}$, *Old*, ... user-specified weights
setLf: *Logical*, ... should Lq, Lf be filled in?
$tdeg \in \mathbf{Z}$, ... total degree
$trL \in \mathbf{R}$, ... trace L
$v \in \mathbf{R}^{n_{vert} \times d}$, *Old*(*nvmax*,*d*), ... vertices
$vc \in \mathbf{Z}$, ... 2^d = number of vertices per cell
$vhit \in \mathbf{Z}^{n_{vert}}$, *Old*(*nvmax*)
$x \in \mathbf{R}^{n \times d}$, *Old*, ... locations of input data
$\xi \in \mathbf{R}^{n_{cell}}$, *Old*(*ncmax*), ... cut value
$y \in \mathbf{R}^{n}$, *Old*

/u3/ehg/a/loess/kd.h
Fri Mar 16 12:45:16 1990

INFO → 1
D → 2
N → 3
VC → 4
NC → 5
NV → 6
*A*1 → 7
*C*1 → 8
*HI*1 → 9
*LO*1 → 10
*V*1 → 11
Ξ1 → 12
*VV*1 → 13
NVMAX → 14
*WORK*1 → 15
*WORK*2 → 16
NCMAX → 17
*WORK*3 → 18
NF → 19
KERNEL → 20
KIND → 21
Π1 → 22
VH → 23
*VV*2 → 24
DIAGL → 25
*WORK*4 → 26
Ψ1 → 27
SEQ → 28
DIM → 29
SING → 30
PRINT → 31
DEG → 32
NDIST → 33
CDEG → 41
F → 1
FCELL → 2
FDIAM → 3
RCOND → 4
Codemacros: ...eqn
Typemacros: *INFO*(*INFO*,1),*D*(*D*,2),*N*(*N*,3),*VC*(*VC*,4),*NC*(*NC*,5),*NV*(*NV*,6),*A*1(*A*1,7),*C*1(*C*1,8),*HI*1(*HI*1
*V*1(*V*1,11),*XI*1(Ξ1,12),*VV*1(*VV*1,13),*NVMAX*(*NVMAX*,14),*WORK*1(*WORK*1,15),*WORK*2(*WORK*2,16),*N*

KIND(*KIND*,21),*PI*1(Π1,22),*VH*(*VH*,23),*VV*2(*VV*2,24),*DIAGL*(*DIAGL*,25),*WORK*4(*WORK*4,26),*PSI*1(Ч
PRINT(*PRINT*,31),*DEG*(*DEG*,32),*NDIST*(*NDIST*,33),*CDEG*(*CDEG*,41),*F*(*F*,1),*FCELL*(*FCELL*,2),*FDIA*

/u3/ehg/a/loess/iv.h
Thu Jan 25 16:17:56 1990

routines

Assert(*%?*,*%ma*,*%i*,*%mi*) → *bold* if (!*%ma*,*ehg*182(*%mi*))
Codemacros: *myAssert*(*Assert*(*%?*,*%ma*,*%i*,*%mi*), *bold* if (!*%ma*,*ehg*182(*%mi*))), ...eqn
Typemacros: ...eqn
bbox≡ehg126
cpvert≡ehg125
descend≡ehg138
eval≡ehg128
l2fit≡ehg136
l2tr≡ehg139
rbuild≡ehg124
spread≡ehg129
vleaf≡ehg137

/u3/ehg/a/loess/routines.h
Fri Mar 9 16:34:12 1990

ehg126

bbox($d,n,vc,x,v,nvmax$)

... fill in vertices for bounding box of x
... lower left, upper right
    for $1 \le k \le d$
        $\alpha := \infty$
        $\beta := -\infty$
        for $1 \le i \le n$
            $t := x_{i,k}$
            $\alpha := \text{Min}(\alpha, t)$
            $\beta := \text{Max}(\beta, t)$
        ... expand the box a little
            $\mu := 0.005 \cdot \text{Max}(\beta - \alpha, 1.e-10 \cdot \text{Max}(|\alpha|, |\beta|) + 1.e-30)$
            $\alpha \mathrel{-}= \mu$
            $\beta \mathrel{+}= \mu$
        $v_{1,k} := \alpha$
        $v_{vc,k} := \beta$
... remaining vertices
    for $2 \le i \le vc - 1$
        $j := i - 1$
        for $1 \le k \le d$
            $v_{i,k} := v_{1+(j \bmod 2)\cdot(vc-1),k}$
            $j := \text{Floor}(j/2)$

$\alpha, \beta \in \mathbf{R}$
$i \in \mathbf{Z}$
$j \in \mathbf{N}$
$k \in \mathbf{Z}$
$\mu, t \in \mathbf{R}$
Include: kd
/u3/ehg/a/loess/bbox.g
Wed Aug 3 12:52:15 1988

ehg125

cpvert($p,n_{vert},v,vhit,nvmax,d,k,t,r,s,$
f,l,u)

```
h := n_vert
for 1≤i≤r
    for 1≤j≤s
        h += 1
        v_h,: := v_f_i,0,j,:
        v_h,k := t
        ... check for redundant vertex
            match := False
            m := 1
            while !match and m≤n_vert
                match := v_m,1 = v_h,1
                mm := 2
                while match and mm≤d
                    match := v_m,mm = v_h,mm
                    mm += 1
                m += 1
            m -= 1
        if match
            h -= 1
        else
            m := h
            if vhit_1 ≥0
                vhit_m := p
        l_i,0,j := f_i,0,j
        l_i,1,j := m
        u_i,0,j := m
        u_i,1,j := f_i,1,j
n_vert := h
Assert(n_vert≤nvmax,180)
```

$f\in \mathbf{Z}^{r\times 0:1\times s}$,*Old*, ... vertex indices of father
$h\in \mathbf{Z}$, ... highest vertex index created so far
$i,j\in \mathbf{Z}$
$k\in \mathbf{Z}$, ... cut axis
$l\in \mathbf{Z}^{r\times 0:1\times s}$,*Old*, ... vertex indices of left son
$m\in \mathbf{Z}$
match: *Logical*
$mm\in \mathbf{Z}$
$p\in \mathbf{Z}$, ... index of father cell

$r,s \in \mathbf{Z}$

$t \in \mathbf{R}$, ... cut

$u \in \mathbf{Z}^{r \times 0:1 \times s}$, *Old*, ... vertex indices of right son

```
Include: kd routines
 /u3/ehg/a/loess/cpvert.g
Thu Sep 15 22:59:57 1988
```

ehg138

j := descend(i,z,a,ξ,lo,hi,$ncmax$)

... descend tree until leaf or ambiguous
$j := i$
while $a_j \neq 0$ **and** $z_{a_j} \neq \xi_j$
    if $z_{a_j} < \xi_j$
        $j := lo_j$
    else
        $j := hi_j$

$a \in \mathbf{Z}^{n_{cell}}, In, Old(ncmax)$, ... cut coordinate (0 for leaf)
$d \in \mathbf{Z}$, ... number of dimensions
$hi \in \mathbf{Z}^{n_{cell}}, In, Old(ncmax)$, ... right subcell or point
$i \in \mathbf{Z}, In$
$j \in \mathbf{Z}$
$lo \in \mathbf{Z}^{n_{cell}}, In, Old(ncmax)$, ... left subcell or point
$\xi \in \mathbf{R}^{n_{cell}}, In, Old(ncmax)$, ... cut value
$z \in \mathbf{R}^{d}, Old(dMAX), In$
Include: kd
/u3/ehg/a/loess/descend.g
Fri Aug 14 09:20:31 1987

ehg106(il,ir,k,nk,p,π,n)

... find the k-th smallest of n elements
... Floyd+Rivest, CACM Mar '75, Algorithm 489
$l := il$
$r := ir$
while $r > l$
    ... to avoid recursion, sophisticated partition deleted
    ... partition $x_{l..r}$ about t
    $t := p_{1,\pi_k}$
    $i := l$
    $j := r$
    Swap(l,k)
    if $p_{1,\pi_r} > t$
        Swap(l,r)
    while $i < j$
        Swap(i,j)
        i += 1
        j −= 1
        while $p_{1,\pi_i} < t$
            i += 1
        while $p_{1,\pi_j} > t$
            j −= 1
    if $p_{1,\pi_l} = t$
        Swap(l,j)
    else
        j += 1
        Swap(r,j)
    if $j \le k$
        $l := j+1$
    if $k \le j$
        $r := j-1$

$Swap(\%?,\%ma,\%?,\%mb) \rightarrow \{ii := \pi_{\%ma},\pi_{\%ma} := \pi_{\%mb},\pi_{\%mb} := ii\}$
$i,il,ir,j \in \mathbf{Z}$
$k \in \mathbf{Z}$, ... select k-th element
$l,n,nk \in \mathbf{Z}$
$p \in \mathbf{R}^{nk \times n}$,$Old$
$\pi \in \mathbf{Z}^n$,Old
$r \in \mathbf{Z}$
$t \in \mathbf{R}$

/u3/ehg/a/loess/ehg106.g

```
ehg127(q,n,d,nf,f,x,ψ,y,rw,kernel,
       k,dist,η,X,od,w,rcond,sing,Σ,U,
       V,γ,qraux,work,tol,dd,tdeg,cdeg,s)

... sort by distance
    dist := 0
    for 1≤j≤dd
        dist += (x_{:,j} − q_j)^2
    PartialSort(1,n,nf,dist,ψ)
    ρ := dist_{ψ_nf} · Max(1.,f)
    Assert(ρ>0,120)
... compute neighborhood weights
    if kernel = 2
        for 1≤i≤nf
            w_i := if(dist_{ψ_i} < ρ, √rw_{ψ_i} ,else,0)
    else
        w := √(dist_ψ / ρ)
        w := √(rw_ψ · (1 − w^3)^3)
    Assert(|| w ||_∞ > 0,121)
... fill design matrix
    column := 1
    X_{:,column} := w
    if tdeg≥1
        for 1≤j≤d
            if cdeg_j≥1
                column += 1
                X_{:,column} := w · (x_{ψ,j} − q_j)
    if tdeg≥2
        for 1≤j≤d
            if cdeg_j≥1
                if cdeg_j≥2
                    column += 1
                    X_{:,column} := w · (x_{ψ,j} − q_j)^2
                for j+1≤jj≤d
                    if cdeg_jj≥1
                        column += 1
                        X_{:,column} := w · (x_{ψ,j} − q_j) · (x_{ψ,jj} − q_jj)
        k := column
    η := w · y_ψ
... equilibrate columns
    for 1≤j≤k
        scal := || X_{:,j} ||_2
        X_{:,j} /= scal
```

```
        colnorm_j := scal
... singular value decomposition
    sqrdc(X,nf,nf,k,qraux,jpvt,work,0)
    sqrsl(X,nf,nf,k,qraux,η,work,η,η,work,
        work,1000,info
    )
    U := 0
    for 1≤i≤k
        for i≤j≤k
            U_{i,j} := X_{i,j}
    ssvdc(U,d2MAX,k,k,Σ,E,U,d2MAX,V,d2MAX,
        work,21,info
    )
    Assert(info = 0,182)
    tol := Σ_1·(100·ε_mach)
    rcond min= Σ_k/Σ_1
    if Σ_k≤tol
        sing += 1
        if sing = 1
            ehg184("Warning. pseudoinverse used at",q,d,1)
            ehg184("neighborhood radius",√ρ,1,1)
            ehg184("reciprocal condition number ",rcond,1,1)
        else if sing = 2
            ehg184("There are other near singularities as well.",ρ,1,1)
... compensate for equilibration
    for 1≤j≤k
        V_{j,:} /= colnorm_j
... solve least squares problem
    for 1≤j≤k
        γ_j := if(Σ_j>tol, <U_{:,j},η> / Σ_j , else, 0.)
    for 0≤j≤od
        s_j := <V_{j+1,:},γ>
```

$E \in \mathbf{R}^{d2MAX}, Old$, ... workspace for svd
$\Sigma \in \mathbf{R}^{k}, Old(d2MAX), Out$
$U \in \mathbf{R}^{k \times k}, Old(d2MAX,d2MAX), Out$, ... upper triangular factor
$V \in \mathbf{R}^{k \times k}, Old(d2MAX,d2MAX), Out$
$X \in \mathbf{R}^{nf \times k}, Old$
$colnorm \in \mathbf{R}^{1:k}, Old(1:d2MAX)$
$column \in \mathbf{Z}$
$dist \in \mathbf{R}^{n}, Old$
$\eta \in \mathbf{R}^{nf}, Old$
$f \in \mathbf{R}, In$

$\gamma \in \mathbf{R}^k, Old(d2MAX), Out,$ ... $\Sigma^+ U^T Q^T W y$
$i, info, j, jj, jpvt \in \mathbf{Z}$
$k \in \mathbf{Z},$ ... dimension of fit 1=const, d+1=linear, (d+2)(d+1)/2=quadratic
$nf \in \mathbf{Z},$ ... number of "near" neighbors
$od \in \mathbf{Z},$ ... 0 means caller wants values only
$q \in \mathbf{R}^d, Old(d),$ ... query point
$qraux \in \mathbf{R}^{d2MAX}, Old, Out$
$rcond \in \mathbf{R}, In, Out,$ ... reciprocal condition number
$\rho \in \mathbf{R}$
$s \in \mathbf{R}^{0:od}, Old,$ ... smoothed values
$scal \in \mathbf{R}$
$sing \in \mathbf{Z}, In, Out,$ ... singularity count
$tol \in \mathbf{R}, Out$
$w \in \mathbf{R}^{nf}, Old,$ ... workspace for regression weights
$work \in \mathbf{R}^{d2MAX}, Old, Out$
$y \in \mathbf{R}^n, Old$
Include: kd routines lin
/u3/ehg/a/loess/ehg127.g
Thu Jul 12 09:22:32 1990

ehg131($x,y,rw,trL,diagL,kernel,k,n,d,n_{cell}$,
$ncmax,vc,n_{vert},nvmax,nf,f,a,c,hi,lo$,
$\pi,\psi,v,vhit,vval,\xi,dist,\eta,X,ntol$,
$f_{diam},w,vval2,rcond,sing,dd,tdeg,cdeg,Lq,Lf$,
$setLf$)

Assert($d \le dMAX$, 101)
... build k-d tree
  bbox($d,n,vc,x,v,nvmax$)
  $n_{vert} := vc$
  $n_{cell} := 1$
  for $1 \le j \le vc$
    $c_{j,n_{cell}} := j$
    $vhit_j := 0$
  $\delta := v_{vc,1:dd} - v_{1,1:dd}$
  $f_{diam} *= ||\delta||_2$
  $\pi := Identity$
  rbuild($1,n,d,n,n_{vert},n_{cell},ncmax,vc,x,\pi$,
    $a,\xi,lo,hi,c,v,vhit,nvmax,ntol,f_{diam}$,
    dd
)
... smooth
  if $trL \neq 0$
    $vval2 := 0$
  $vval$:= l2tr($v,nvmax,n_{vert},n,d,nf,f,x,\pi,\psi$,
    $y,rw,trL,kernel,k,dist,dist,\eta,X,d$,
    $w,diagL,vval2,n_{cell},vc,a,\xi,lo,hi,c$,
    $vhit,rcond,sing,dd,tdeg,cdeg,Lq,Lf,setLf$
)

$X \in \mathbf{R}^{\cdot}, Old$
bbox≡ehg126
$\delta \in \mathbf{R}^d, Old(dMAX)$
$diagL \in \mathbf{R}^n, Old$
$dist \in \mathbf{R}^n, Old$
$f \in \mathbf{R}, In$
$j,k \in \mathbf{Z}$
$nf \in \mathbf{Z}$, ... number of "near" neighbors
$ntol \in \mathbf{Z}$
rbuild≡ehg124
$rcond \in \mathbf{R}, Out$, ... reciprocal condition number
$sing \in \mathbf{Z}, In, Out$
$vval \in \mathbf{R}^{0:d \times n_{vert}}, Old(0:d, nvmax)$, ... smoothed values at vertices
$vval2 \in \mathbf{R}^{0:d \times n_{vert}}, Old(0:dMAX, nvmax)$

No! Should be d.

$w \in \mathbf{R}^{nf}$,*Old*, ... workspace for l2fit
$y \in \mathbf{R}^{n}$, ... observations ,*Old*
Include: kd routines approx lin
/u3/ehg/a/loess/ehg131.g
Sat Mar 17 10:33:58 1990

ehg133($n,d,vc,nvmax,n_{cell},ncmax,a,c,hi,lo,$
$v,vval,\xi,m,z,s$)

for $1 \le i \le m$
$\delta := z_{i,:}$
$s_i :=$ eval($\delta,d,ncmax,vc,a,\xi,lo,hi,c,v,$
$nvmax,vval$
)

$\delta \in \mathbf{R}^d, Old(dMAX)$
$i,m \in \mathbf{Z}$
$s \in \mathbf{R}^m, Old$
$vval \in \mathbf{R}^{0:d \times n_{vert}}, Old(0{:}d,nvmax)$, ... smoothed values at vertices
$z \in \mathbf{R}^{m \times d}, Old$
Include: kd routines approx lin
/fs/pyxis6/ehg/a/loess/ehg133.g
Wed May 29 14:00:58 1991

ehg140(iw,i,j)

$iw_i := j$

$i \in \mathbf{Z}$
$iw \in \mathbf{Z}^i$,*Old*,*In*,*Out*
$j \in \mathbf{Z}$

/u3/ehg/a/loess/ehg140.g
Wed Aug 12 14:10:08 1987

ehg169($d, vc, n_{cell}, ncmax, n_{vert}, nvmax, v, a, \xi, c,$
hi, lo)

... as in bbox
... remaining vertices
for $2 \le i \le vc-1$
$j := i-1$
for $1 \le k \le d$
$v_{i,k} := v_{1+(j \bmod 2)\cdot(vc-1),k}$
$j := \text{Floor}(j/2)$
... as in ehg131
$m_{cell} := 1$
$m_{vert} := vc$
$novhit_1 := -1$
for $1 \le j \le vc$
$c_{j,m_{cell}} := j$
... as in rbuild
$p := 1$
while $p \le n_{cell}$
if $a_p \neq 0$
$k := a_p$
... left son
$m_{cell} \mathrel{+}= 1$
$lo_p := m_{cell}$
... right son
$m_{cell} \mathrel{+}= 1$
$hi_p := m_{cell}$
cpvert($p, m_{vert}, v, novhit, nvmax, d, k, \xi_p, 2^{k-1}, 2^{d-k}$,
$c_{:,p}, c_{:,lo_p}, c_{:,hi_p}$
)
$p \mathrel{+}= 1$
Assert($m_{cell} = n_{cell}$,193)
Assert($m_{vert} = n_{vert}$,193)

$i, j, k \in \mathbf{Z}$
$m_{cell} \in \mathbf{Z}$, ... local version of n_{cell}
$m_{vert} \in \mathbf{Z}$, ... local version of n_{vert}
$novhit \in \mathbf{Z}^1, Old$, ... dummy
$p \in \mathbf{Z}$, ... cell index
Include: kd routines
/u3/ehg/a/loess/ehg169.g
Thu Sep 15 23:04:06 1988

ehg170($k,d,vc,n_{vert},nvmax,n_{cell},ncmax,a,c,hi,$
$lo,v,vval,\xi$)

```
Out("     real function loeval(z)")
Out("     real z(*)")
Out("     integer d,vc,nv,nc")
Out("     integer a(",n_cell,"), c(",vc,",",n_cell,")")
Out("     integer hi(",n_cell,"), lo(",n_cell,")")
Out("     real v(",n_vert,",",d,")")
Out("     real vval(0:",d,",",n_vert,")")
Out("     real xi(",n_cell,")")
Out("     real ehg128")
Out("     data d,vc,nv,nc /",d,",",vc,",",n_vert,",",n_cell,"/")
for  1≤i≤n_cell
   Out("     data a(",i,") /",a_i,"/")
   if  a_i≠0
      Out("     data hi(",i,"),lo(",i,"),xi(",i,") /",hi_i,",",lo_i,
         ",",ξ_i,"/"
      )
   for  1≤j≤vc
      Out("     data c(",j,",",i,") /",c_{j,i},"/")
for  1≤i≤n_vert
   Out("     data vval(0,",i,") /",vval_{0,i},"/")
   for  1≤j≤d
      Out("     data v(",i,",",j,") /",v_{i,j},"/")
      Out("     data vval(",j,",",i,") /",vval_{j,i},"/")
Out("     loeval=ehg128(z,d,nc,vc,a,xi,lo,hi,c,v,nv,vval)")
Out("     end")
```

$i,j \in \mathbf{Z}$
$vval \in \mathbf{R}^{0:d \times nvmax}, Old$
Include: kd
/u3/ehg/a/loess/ehg170.g
Thu Sep 15 08:59:07 1988

ehg191($m,z,L,d,n,nf,n_{vert},ncmax,vc,a,$
$\xi,lo,hi,c,v,nvmax,vval2,Lf,Lq$)

for $1 \le j \le n$
  $vval2 := 0$
  for $1 \le i \le n_{vert}$
    ... linear search for i in Lq
      $Lq1 := Lq_{i,1}$
      $Lq_{i,1} := j$
      $p := nf$
      while $Lq_{i,p} \ne j$
        $p -= 1$
      $Lq_{i,1} := Lq1$
    if $Lq_{i,p} = j$
      $vval2_{:,i} := Lf_{:,i,p}$
  for $1 \le i \le m$
    $zi := z_{i,:}$
    $L_{i,j} :=$ eval($zi,d,ncmax,vc,a,\xi,lo,hi,c,v,$
      $nvmax,vval2$
)

$L \in \mathbf{R}^{m \times n}, Old$
$Lq1,i,j,m,p \in \mathbf{Z}$
$vval2 \in \mathbf{R}^{0:d \times n_{vert}}, Old(0:d,nvmax)$
$z \in \mathbf{R}^{m \times d}, Old$
$zi \in \mathbf{R}^{d}, Old(dMAX)$ 22 Jun 92
Include: kd routines
/u3/ehg/a/loess/ehg191.g
Wed Apr 4 23:41:24 1990

ehg192$(y,d,n,nf,n_{vert},nvmax,vval,Lf,Lq)$

$vval := 0$
for $1 \le i \le n_{vert}$
  for $1 \le j \le nf$
    $vval_{:,i} \mathrel{+}= y_{Lq_{i,j}} \cdot Lf_{:,i,j}$

$i,j \in \mathbf{Z}$
$vval \in \mathbf{R}^{0:d \times n_{vert}}, Old(0{:}d, nvmax)$
Include: kd routines
/u3/ehg/a/loess/ehg192.g
Tue Apr 3 21:31:35 1990

```
ehg128

s := eval(z,d,ncmax,vc,a,ξ,lo,hi,c,v,
      nvmax,vval)

... locate enclosing cell
    nt := 1
    t_nt := 1
    j := 1
    while a_j≠0
        ++nt
        t_nt := if(z_{a_j}<ξ_j,lo_j,else,hi_j)
        Assert(nt<20,181)
        j := t_nt
... tensor
    g := vval_{:,c_{:,j}}
    lg := vc
    ll := c_{1,j}
    ur := c_{vc,j}
    for d≥i≥1
        h := (z_i − v_{ll,i}) / (v_{ur,i} − v_{ll,i})
        if h<−.001
            ehg184("eval ",z,d,1)
            ehg184("lowerlimit ",v_{ll,:},d,nvmax)
        else if h>1.001
            ehg184("eval ",z,d,1)
            ehg184("upperlimit ",v_{ur,:},d,nvmax)
        Assert(−.001≤h and h≤1.001,122)
        lg := Floor(lg/2)
        for 1≤ig≤lg
            ... Hermite basis
                φ0 := (1−h)^2·(1+2·h)
                φ1 := h^2·(3−2·h)
                ψ0 := h·(1−h)^2
                ψ1 := h^2·(h−1)
            g_{0,ig} := φ0·g_{0,ig} + φ1·g_{0,ig+lg} + (ψ0·g_{i,ig} + ψ1·g_{i,ig+lg})·(v_{ur,i} − v_{ll,i})
            for 1≤ii≤i−1
                g_{ii,ig} := φ0·g_{ii,ig} + φ1·g_{ii,ig+lg}
    s := g_{0,1}
... blending
    if d = 2
        ... ----- North -----
            v0 := v_{ll,1}
```

$v1 := v_{ur,1}$
$g0 := vval_{:,c_{3,j}}$
$g1 := vval_{:,c_{4,j}}$
$\bar{\xi} := v_{ur,2}$
$m := nt-1$
until ($m = 0$ **or** ($a_{t_m} = 2$ **and** $\xi_{t_m} = \bar{\xi}$))
  $--m$
if $m \geq 1$
  $m := hi_{t_m}$
  while $a_m \neq 0$
    if $z_{a_m} < \xi_m$
      $m := lo_m$
    else
      $m := hi_m$
  if $v_{c_{1,m},1} > v0$
    $v0 := v_{c_{1,m},1}$
    $g0 := vval_{:,c_{1,m}}$
  if $v_{c_{2,m},1} < v1$
    $v1 := v_{c_{2,m},1}$
    $g1 := vval_{:,c_{2,m}}$
$h := \frac{z_1 - v0}{v1 - v0}$
... Hermite basis
  $\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$
  $\phi 1 := h^2 \cdot (3-2 \cdot h)$
  $\psi 0 := h \cdot (1-h)^2$
  $\psi 1 := h^2 \cdot (h-1)$
$gN := \phi 0 \cdot g0_0 + \phi 1 \cdot g1_0 + (\psi 0 \cdot g0_1 + \psi 1 \cdot g1_1) \cdot (v1 - v0)$
$gpN := \phi 0 \cdot g0_2 + \phi 1 \cdot g1_2$
... ----- South -----
$v0 := v_{ll,1}$
$v1 := v_{ur,1}$
$g0 := vval_{:,c_{1,j}}$
$g1 := vval_{:,c_{2,j}}$
$\bar{\xi} := v_{ll,2}$
$m := nt-1$
until ($m = 0$ **or** ($a_{t_m} = 2$ **and** $\xi_{t_m} = \bar{\xi}$))
  $--m$
if $m \geq 1$
  $m := lo_{t_m}$
  while $a_m \neq 0$
    if $z_{a_m} < \xi_m$

$m := lo_m$

else

$m := hi_m$

if $v_{c_{3,m},1} > v0$

$v0 := v_{c_{3,m},1}$

$g0 := vval_{:,c_{3,m}}$

if $v_{c_{4,m},1} < v1$

$v1 := v_{c_{4,m},1}$

$g1 := vval_{:,c_{4,m}}$

$h := \dfrac{z_1 - v0}{v1 - v0}$

... Hermite basis

$\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$

$\phi 1 := h^2 \cdot (3-2 \cdot h)$

$\psi 0 := h \cdot (1-h)^2$

$\psi 1 := h^2 \cdot (h-1)$

$gS := \phi 0 \cdot g0_0 + \phi 1 \cdot g1_0 + (\psi 0 \cdot g0_1 + \psi 1 \cdot g1_1) \cdot (v1 - v0)$

$gpS := \phi 0 \cdot g0_2 + \phi 1 \cdot g1_2$

... ----- East -----

$v0 := v_{ll,2}$

$v1 := v_{ur,2}$

$g0 := vval_{:,c_{2,j}}$

$g1 := vval_{:,c_{4,j}}$

$\overline{\xi} := v_{ur,1}$

$m := nt - 1$

until ($m = 0$ **or** ($a_{t_m} = 1$ **and** $\xi_{t_m} = \overline{\xi}$))

$--m$

if $m \geq 1$

$m := hi_{t_m}$

while $a_m \neq 0$

if $z_{a_m} < \xi_m$

$m := lo_m$

else

$m := hi_m$

if $v_{c_{1,m},2} > v0$

$v0 := v_{c_{1,m},2}$

$g0 := vval_{:,c_{1,m}}$

if $v_{c_{3,m},2} < v1$

$v1 := v_{c_{3,m},2}$

$g1 := vval_{:,c_{3,m}}$

$h := \dfrac{z_2 - v0}{v1 - v0}$

... Hermite basis

$\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$

$\phi 1 := h^2 \cdot (3-2 \cdot h)$

$\psi 0 := h \cdot (1-h)^2$

$\psi 1 := h^2 \cdot (h-1)$

$gE := \phi 0 \cdot g0_0 + \phi 1 \cdot g1_0 + (\psi 0 \cdot g0_2 + \psi 1 \cdot g1_2) \cdot (v1 - v0)$

$gpE := \phi 0 \cdot g0_1 + \phi 1 \cdot g1_1$

... ----- West -----

$v0 := v_{ll,2}$

$v1 := v_{ur,2}$

$g0 := vval_{:,c_{1,j}}$

$g1 := vval_{:,c_{3,j}}$

$\overline{\xi} := v_{ll,1}$

$m := nt - 1$

until ($m = 0$ **or** ($a_{t_m} = 1$ **and** $\xi_{t_m} = \overline{\xi}$))

$--m$

if $m \geq 1$

$m := lo_{t_m}$

while $a_m \neq 0$

if $z_{a_m} < \xi_m$

$m := lo_m$

else

$m := hi_m$

if $v_{c_{2,m},2} > v0$

$v0 := v_{c_{2,m},2}$

$g0 := vval_{:,c_{2,m}}$

if $v_{c_{4,m},2} < v1$

$v1 := v_{c_{4,m},2}$

$g1 := vval_{:,c_{4,m}}$

$h := \dfrac{z_2 - v0}{v1 - v0}$

... Hermite basis

$\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$

$\phi 1 := h^2 \cdot (3-2 \cdot h)$

$\psi 0 := h \cdot (1-h)^2$

$\psi 1 := h^2 \cdot (h-1)$

$gW := \phi 0 \cdot g0_0 + \phi 1 \cdot g1_0 + (\psi 0 \cdot g0_2 + \psi 1 \cdot g1_2) \cdot (v1 - v0)$

$gpW := \phi 0 \cdot g0_1 + \phi 1 \cdot g1_1$

... NS

$h := \dfrac{z_2 - v_{ll,2}}{v_{ur,2} - v_{ll,2}}$

... Hermite basis

$\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$

$\phi 1 := h^2 \cdot (3 - 2 \cdot h)$

$\psi 0 := h \cdot (1-h)^2$

$\psi 1 := h^2 \cdot (h-1)$

$s_{NS} := \phi 0 \cdot gS + \phi 1 \cdot gN + (\psi 0 \cdot gpS + \psi 1 \cdot gpN) \cdot (v_{ur,2} - v_{ll,2})$

... EW

$h := \dfrac{z_1 - v_{ll,1}}{v_{ur,1} - v_{ll,1}}$

... Hermite basis

$\phi 0 := (1-h)^2 \cdot (1+2 \cdot h)$

$\phi 1 := h^2 \cdot (3-2 \cdot h)$

$\psi 0 := h \cdot (1-h)^2$

$\psi 1 := h^2 \cdot (h-1)$

$s_{EW} := \phi 0 \cdot gW + \phi 1 \cdot gE + (\psi 0 \cdot gpW + \psi 1 \cdot gpE) \cdot (v_{ur,1} - v_{ll,1})$

$s := (s_{NS} + s_{EW}) - s$

$g \in \mathbf{R}^{0:d \times vc}, Old(0{:}dMAX, vcMAX)$, ... *vval* workspace

$g0 \in \mathbf{R}^{0:d}, Old(0{:}dMAX)$

$g1 \in \mathbf{R}^{0:d}, Old(0{:}dMAX)$

$gE, gN, gS, gW, gpE, gpN, gpS, gpW, h \in \mathbf{R}$

$i, ig, ii \in \mathbf{Z}$

$j \in \mathbf{Z}$, ... enclosing cell

$lg \in \mathbf{N}$, ... active length of g

$ll \in \mathbf{Z}$, ... lower left vertex

$m \in \mathbf{Z}$, ... going up: depth in stack; going down: cell

$nt \in \mathbf{Z}$, ... top of cell stack

$phi0, phi1, psi0, psi1, s, s_{EW}, s_{NS} \in \mathbf{R}$

$t \in \mathbf{Z}^{20}, Old$, ... stack of cell from root to leaf

$ur \in \mathbf{Z}$, ... upper right vertex

$v0, v1 \in \mathbf{R}$

$vval \in \mathbf{R}^{0:d \times nvmax}, Old$, ... vertex values

$\bar{\xi} \in \mathbf{R}$, ... cut-value forming cell side

$z \in \mathbf{R}^d, Old$

```
Include: kd routines
 /u3/ehg/a/loess/eval.g
Thu Jan 25 16:27:38 1990
```

```
ehg136

s := l2fit(u,lm,m,n,d,nf,f,x,ψ,y,
      rw,kernel,k,dist,η,X,od,L,ihat,w,
      rcond,sing,dd,tdeg,cdeg)

Assert(k≤nf−1,104)
Assert(k≤d2MAX,105)
ψ := Identity
for 1≤l≤m
    q := u_{l,:}
    ehg127(q,n,d,nf,f,x,ψ,y,rw,kernel,
        k,dist,η,X,od,w,rcond,sing,Σ,U,
        V,γ,qraux,work,tol,dd,tdeg,cdeg,s_{:,l}
    )
    if ihat = 1  ... L_{l,l}=V_{1,:}Σ^+ U^T(Q^T We_i)
        Assert(m = n,123)
        ... find i such that l=ψ_i
            i := 1
            while l≠ψ_i
                i += 1
                Assert(i<nf,123)
        η := 0
        η_i := w_i
        ... η=Q^T We_i
            sqrsl(X,nf,nf,k,qraux,η,η,η,η,η,
                η,1000,info
            )
        ... γ=U^T η_{1:k}
            γ := 0
            for 1≤j≤k
                γ += η_j·U_{j,:}
        ... γ=Σ^+ γ
            for 1≤j≤k
                if Σ_j>tol
                    γ_j /= Σ_j
                else
                    γ_j := 0.
        L_{l,1} := <V_{1,:},γ>
    else if ihat = 2  ... L_{l,:}=V_{1,:}Σ^+ (U^T Q^T)W
        L_{l,:} := 0
        for 1≤j≤k
            η := 0
            η_{1:k} := U_{:,j}
```

```
sqrsl(X,nf,nf,k,qraux,η,η,work,work,work,
    work,10000,info
)
if  Σ_j > tol
    scale := 1./Σ_j
else
    scale := 0.
η *= scale·w
for  1≤i≤nf
    L_{l,ψ_i} += V_{1,j}·η_i
```

$L \in \mathbf{R}^{m \times n}, Old, Out,$ ... if hat=1, dimension n by 1
$\Sigma \in \mathbf{R}^{k}, Old(d2MAX)$
$U \in \mathbf{R}^{k \times k}, Old(d2MAX, d2MAX),$ ... upper triangular factor
$V \in \mathbf{R}^{k \times k}, Old(d2MAX, d2MAX)$
$X \in \mathbf{R}^{nf \times k}, Old$
$dist \in \mathbf{R}^{n}, Old$
$\eta \in \mathbf{R}^{nf}, Old$
$f \in \mathbf{R}, In$
$\gamma \in \mathbf{R}^{k}, Old(d2MAX),$ ... $\Sigma^{+} U^{T} Q^{T} W y$
$i \in \mathbf{Z}$
$ihat \in \mathbf{Z}, In$
$info, j \in \mathbf{Z}$
$k \in \mathbf{Z},$ ... dimension of fit 1=const, d+1=linear, (d+2)(d+1)/2=quadratic
$l, lm, m \in \mathbf{Z}$
$nf \in \mathbf{Z},$ ... number of "near" neighbors
$od \in \mathbf{Z},$ ... 0 means caller wants values only
$q \in \mathbf{R}^{d}, Old(dMAX),$ ... u(1,:)
$qraux \in \mathbf{R}^{d2MAX}, Old$
$rcond \in \mathbf{R}, In, Out,$ ... reciprocal condition number
$s \in \mathbf{R}^{0:od \times m}, Old,$ ... smoothed values
$scale \in \mathbf{R}$
$sing \in \mathbf{Z}, In, Out,$ ... singularity count
$tol \in \mathbf{R}$
$u \in \mathbf{R}^{m \times d}, Old(lm, d),$ ... query points
$w \in \mathbf{R}^{nf}, Old,$ ... workspace for regression weights
$work \in \mathbf{R}^{d2MAX}, Old$
$y \in \mathbf{R}^{n}, Old$
Include: kd routines lin
 /fs/pyxis6/ehg/a/loess/l2fit.g
Wed May 29 14:30:59 1991

ehg139

s := l2tr($v, nvmax, n_{vert}, n, d, nf, f, x, \pi, \psi,$
$y, rw, trL, kernel, k, dist, \phi, \eta, X, od,$
$w, diagL, vval2, ncmax, vc, a, \xi, lo, hi, c,$
$vhit, rcond, sing, dd, tdeg, cdeg, Lq, Lf, setLf$)

... l2fit with trace(L)
Assert($k \le nf-1$,104)
Assert($k \le d2MAX$,105)
if $trL \neq 0$
  $diagL := 0$
  $vval2 := 0$
$\psi := Identity$
for $1 \le l \le n_{vert}$
  $q := v_{l,:}$
  ehg127($q, n, d, nf, f, x, \psi, y, rw, kernel,$
    $k, dist, \eta, X, od, w, rcond, sing, \Sigma, U,$
    $V, \gamma, qraux, work, tol, dd, tdeg, cdeg, s_{:,l}$
)
  if $trL \neq 0$
    ... invert ψ
      $\phi := 0$
      for $1 \le i \le nf$
        $\phi_{\psi_i} := i$
    $z := v_{l,:}$
    vleaf($z, vhit_l, leaf, nleaf, d, n_{vert}, nvmax, ncmax, vc, a,$
      ξ, lo, hi, c, v
)
    for $1 \le ileaf \le nleaf$
      for $lo_{leaf_{ileaf}} \le ii \le hi_{leaf_{ileaf}}$
        $i := \phi_{\pi_{ii}}$
        if $i \neq 0$
          Assert($\psi_i = \pi_{ii}$,194)
          $\eta := 0$
          $\eta_i := w_i$
          ... $\eta = Q^T W e_i$
          sqrsl($X, nf, nf, k, qraux, \eta, work, \eta, \eta, work,$
            $work$,1000,$info$
)
          for $1 \le j \le k$
            $\gamma_j :=$ **if**$(\Sigma_j > tol, \frac{<U_{:,j}, \eta>}{\Sigma_j}$, **else**, 0.)

```
                for  1≤j≤d+1
                    vval2_{j-1,l} := <V_{j,:},γ>
                z := x_{π_ii,:}
                term := eval(z,d,ncmax,vc,a,ξ,lo,hi,c,v,
                    nvmax,vval2
                )
                diagL_{π_ii} += term
                vval2_{:,l} := 0
    if  setLf  ... like l2fit; Lf_:,l,:=Vgma +U TQ TW
        Assert(k≥d+1,196)
        Lq_{l,:} := ψ_{1:nf}
        Lf_{:,l,:} := 0
        for  1≤j≤k
            η := 0
            η_{1:k} := U_{:,j}
            sqrsl(X,nf,nf,k,qraux,η,η,work,work,work,
                work,10000,info
            )
            if  Σ_j>tol
                scale := 1./Σ_j
            else
                scale := 0.
            η *= scale·w
            for  1≤i≤nf
                Lf_{:,l,i} += V_{1:d+1,j}·η_i
if  trL≠0
    trL := Sum(diagL)
```

$\Sigma \in \mathbf{R}^k$, *Old*(*d2MAX*)
$U \in \mathbf{R}^{k \times k}$, *Old*(*d2MAX*,*d2MAX*), ... upper triangular factor
$V \in \mathbf{R}^{k \times k}$, *Old*(*d2MAX*,*d2MAX*)
$X \in \mathbf{R}^{nf \times k}$, *Old*
$diagL \in \mathbf{R}^n$, *Old*
$dist \in \mathbf{R}^n$, *Old*, ... may overlay ϕ
$\eta \in \mathbf{R}^{nf}$, *Old*
$f \in \mathbf{R}$, *In*
$\gamma \in \mathbf{R}^k$, *Old*(*d2MAX*), ... $\Sigma^+ U^T Q^T W y$
$i, ii, ileaf, info, j, k, l \in \mathbf{Z}$
$leaf \in \mathbf{Z}^{vcMAX}$, *Old*
$n_{vert} \in \mathbf{Z}$, ... number of vertices
$nf \in \mathbf{Z}$, ... number of "near" neighbors
$nleaf \in \mathbf{Z}$
$nvmax \in \mathbf{Z}$, ... allocated size of v

$od \in \mathbf{Z}$, ... 0 means caller wants values only
$\phi \in \mathbf{Z}^n, Permutation, Old$, ... may overlay *dist*
$q \in \mathbf{R}^d, Old(dMAX), In$, ... query point
$qraux \in \mathbf{R}^{d2MAX}, Old$
$rcond \in \mathbf{R}, In, Out$, ... reciprocal condition number
$s \in \mathbf{R}^{0:od \times n_{vert}}, Old$, ... smoothed values
$scale \in \mathbf{R}$
$sing \in \mathbf{Z}, In, Out$
$term \in \mathbf{R}$, ... contribution to diag L
$tol \in \mathbf{R}$
$vval2 \in \mathbf{R}^{0:d \times n_{vert}}, Old$
$w \in \mathbf{R}^{nf}, Old$, ... workspace for regression weights
$work \in \mathbf{R}^{d2MAX}, Old$
$y \in \mathbf{R}^n, Old$
$z \in \mathbf{R}^d, Old(dMAX), dMAX$
Include: kd lin routines
/u3/ehg/a/loess/l2tr.g
Sat Mar 17 13:45:23 1990

lofort(*iunit*,*iv*,*liv*,*lv*,*wv*)

ehg170(*iunit*, iv_D, iv_{VC}, iv_{NV}, iv_{NVMAX}, iv_{NC}, iv_{NCMAX}, $iv_{iv_{A1}}$, $iv_{iv_{C1}}$, $iv_{iv_{H1}}$,
$iv_{iv_{LO1}}$, $wv_{iv_{V1}}$, $wv_{iv_{VV1}}$, $wv_{iv_{S1}}$
)

---

iunit $\in \mathbf{Z}$,*In*, ... Fortran unit number for generated Fortran
iv $\in \mathbf{Z}^{\cdot}$,*Old*
wv $\in \mathbf{R}^{\cdot}$,*Old*
Include: kd routines iv approx lin
/u3/ehg/a/loess/lofort.g
Sun Jun 12 10:39:27 1988

loread($iunit, d, vc, nc, nv, iv, liv, lv, v$)

$iv_{SEQ} := 173$
$iv_D := d$
$iv_{VC} := vc$
$iv_{NVMAX} := nv$
$iv_{NCMAX} := nc$
$iv_{A1} := 50$
$iv_{C1} := iv_{A1} + nc$
$iv_{HI1} := iv_{C1} + vc \cdot nc$
$iv_{LO1} := iv_{HI1} + nc$
$bound := iv_{LO1} + nc$
Assert($bound - 1 \le liv, 102$)
$iv_{V1} := 50$
$iv_{VV1} := iv_{V1} + nv \cdot d$
$iv_{\Xi 1} := iv_{VV1} + (d+1) \cdot nv$
$bound := iv_{\Xi 1} + nc$
Assert($bound - 1 \le lv, 103$)
ehg168($iunit, d, vc, nc, nv, nv, v_{iv_{V1}}, iv_{iv_{A1}}, v_{iv_{\Xi 1}}, v_{iv_{VV1}}$)
ehg169($d, vc, nc, nc, nv, nv, v_{iv_{V1}}, iv_{iv_{A1}}, v_{iv_{\Xi 1}}, iv_{iv_{C1}}$,
$iv_{iv_{HI1}}, iv_{iv_{LO1}}$
)

$bound \in \mathbf{Z}$
$d \in \mathbf{Z}, In$
$iunit \in \mathbf{Z}, In,$ ... Fortran unit to read from
$iv \in \mathbf{Z}^{liv}, Old, Out$
$liv \in \mathbf{Z}, In$
$lv \in \mathbf{Z}, In$
$nc \in \mathbf{Z}, In$
$nv \in \mathbf{Z}, In$
$v \in \mathbf{R}^{lv}, Old, Out$
$vc \in \mathbf{Z}, In$
Include: iv routines
/fs/pyxis6/ehg/a/loess/loread.g
Thu Sep 15 23:10:08 1988

losave(*iunit*,*iv*,*liv*,*lv*,*v*)

ehg167($iunit, iv_D, iv_{VC}, iv_{NC}, iv_{NV}, iv_{NVMAX}, v_{iv_{V1}}, iv_{iv_{A1}}, v_{iv_{\Sigma 1}}, v_{iv_{VV1}}$)

---

iunit$\in \mathbf{Z}$,*In*, ... Fortran unit to write on
iv$\in \mathbf{Z}^{liv}$,*Old*,*In*
liv$\in \mathbf{Z}$,*In*
lv$\in \mathbf{Z}$,*In*
v$\in \mathbf{R}^{lv}$,*Old*,*In*
Include: iv
/u3/ehg/a/loess/losave.g
Sun Jun 12 09:51:48 1988

lowesa($trl,n,d,\tau,nsing,\delta_1,\delta_2$)

ehg141($trl,n,1,\tau,d,nsing,dka,d1a,d2a$)
ehg141($trl,n,2,\tau,d,nsing,dkb,d1b,d2b$)
$\alpha := \frac{\tau - dka}{dkb - dka}$
$\delta_1 := (1-\alpha)\cdot d1a + \alpha\cdot d1b$
$\delta_2 := (1-\alpha)\cdot d2a + \alpha\cdot d2b$

$\alpha \in \mathbf{R}$
$d \in \mathbf{Z}, In$
$d1a, d1b, d2a, d2b \in \mathbf{R}$
$\delta_1 \in \mathbf{R}, Out$
$\delta_2 \in \mathbf{R}, Out$
$dka, dkb \in \mathbf{Z}$
$n \in \mathbf{Z}, In$
$nsing \in \mathbf{Z}, In$
$\tau \in \mathbf{Z}, In$, ... degrees of freedom in local model
$trl \in \mathbf{R}, In$
Include: iv routines
/fs/pyxis6/ehg/a/loess/lowesa.g
Wed Jun 5 02:26:44 1991

lowesb(xx,yy,ww,$diagL$,$infl$,iv,liv,lv,wv)

Assert($iv_{SEQ} \neq 173$ and $iv_{SEQ} \neq 174$, 174)
if $iv_{SEQ} \neq 172$
    Assert($iv_{SEQ} = 171$,171)
$iv_{SEQ} := 173$
$trL :=$ **if**($infl$,1.,**else**,0.)
$setLf := iv_{\Psi 1} \neq iv_{LQ}$
ehg131(xx,yy,ww,trL,$diagL$,iv_{KERNEL},iv_{DIM},iv_N,iv_D,iv_{NC},
    iv_{NCMAX},iv_{VC},iv_{NV},iv_{NVMAX},iv_{NF},wv_F,$iv_{iv_{A1}}$,$iv_{iv_{C1}}$,$iv_{iv_{H11}}$,$iv_{iv_{LO1}}$,
    $iv_{iv_{\Pi 1}}$,$iv_{iv_{\Psi 1}}$,$wv_{iv_{V1}}$,$iv_{iv_{VH}}$,$wv_{iv_{VV1}}$,$wv_{iv_{g1}}$,$wv_{iv_{WORK1}}$,$wv_{iv_{WORK2}}$,$wv_{iv_{WORK3}}$,Floor($iv_N \cdot wv_{FCELL}$),
    wv_{FDIAM},$wv_{iv_{WORK4}}$,$wv_{iv_{VV2}}$,wv_{RCOND},iv_{SING},iv_{NDIST},iv_{DEG},$iv_{CDEG:CDEG+8}$,$iv_{iv_{LQ}}$,$wv_{iv_{LF}}$,
    $setLf$
)
if $iv_{NV} + iv_{VC}/2 > iv_{NVMAX}$
    ehg183("Warning. k-d tree limited by memory; nvmax=",iv_{NVMAX},1,1)
else if $iv_{NC} + 2 > iv_{NCMAX}$
    ehg183("Warning. k-d tree limited by memory. ncmax=",iv_{NCMAX},1,1)

$diagL \in \mathbf{R}^{\cdot}$,*Old*,*Out*
infl: *Logical*,*In*
$iv \in \mathbf{Z}^{\cdot}$,*Old*
setLf: *Logical*, ... should Lq, Lf be filled in?
$wv \in \mathbf{R}^{\cdot}$,*Old*
$ww \in \mathbf{R}^{\cdot}$,*Old*
$xx \in \mathbf{R}^{\cdot}$,*Old*,*Matrix*
$yy \in \mathbf{R}^{\cdot}$,*Old*
Include: kd routines iv approx lin
/u3/ehg/a/loess/lowesb.g
Sat Mar 17 13:25:14 1990

lowesc(n,L,LL,trL,δ_1,δ_2)

... compute $LL = (I-L)(I-L)'$
    for $1 \le i \le n$
        $L_{i,i}$ -= 1
    for $1 \le i \le n$
        for $1 \le j \le i$
            $LL_{i,j} := \langle L_{i,:}, L_{j,:} \rangle$
    for $1 \le i \le n$
        for $i+1 \le j \le n$
            $LL_{i,j} := LL_{j,i}$
    for $1 \le i \le n$
        $L_{i,i}$ += 1
... accumulate first two traces
    $trL := 0$
    $\delta_1 := 0$
    for $1 \le i \le n$
        trL += $L_{i,i}$
        δ_1 += $LL_{i,i}$
... $\delta_2 = trLL^2$
    $\delta_2 := 0$
    for $1 \le i \le n$
        δ_2 += $\langle LL_{i,:}, LL_{:,i} \rangle$

$L \in \mathbf{R}^{n \times n}$,*Old*,*In*
$LL \in \mathbf{R}^{n \times n}$,*Old*,*Out*
$\delta_1 \in \mathbf{R}$,*Out*, ... $tr(I-L)(I-L)'$
$\delta_2 \in \mathbf{R}$,*Out*, ... $tr[(I-L)(I-L)']^2$
$i,j \in \mathbf{Z}$
$n \in \mathbf{Z}$,*In*
$trL \in \mathbf{R}$,*Out*
Include: lin
 /fs/pyxis6/ehg/a/loess/lowesc.g
Thu Oct 22 17:03:18 1987

lowesd(*version*,*iv*,*liv*,*lv*,*v*,*d*,*n*,*f*,*ideg*,*setLf*)

Assert(*version* = ~~105~~,100)
$iv_{SEQ} := 171$
$iv_D := d$
$iv_N := n$
$vc := 2^d$
$iv_{VC} := vc$
Assert($f>0$,120)
$nf := \text{Min}(n,\text{Floor}(n \cdot f))$
$iv_{NF} := nf$
$iv_{KERNEL} := 1$
$iv_{DIM} :=$ **if**$(ideg = 0,1,ideg = 1,d+1,ideg = 2,\frac{(d+2)\cdot(d+1)}{2})$
$iv_{KIND} := 1$
~~$nvmax := \text{Max}(200,n)$~~
$iv_{NVMAX} := nvmax$
$ncmax :=$ ~~$\text{Max}(200,n)$~~
$iv_{NCMAX} := ncmax$
$iv_{SING} := 0$
$iv_{DEG} := ideg$
Assert($ideg \geq 0$,195)
Assert($ideg \leq 2$,195)
$iv_{NDIST} := d$
$iv_{CDEG:CDEG+8} := ideg$
$iv_{A1} := 50$
$iv_{C1} := iv_{A1} + ncmax$
$iv_{HI1} := iv_{C1} + vc \cdot ncmax$
$iv_{LO1} := iv_{HI1} + ncmax$
$iv_{\Pi 1} := iv_{LO1} + ncmax$
... initialize permutation
    $j := iv_{\Pi 1} - 1$
    for $1 \leq i \leq n$
        $iv_{j+i} := i$
$iv_{VH} := iv_{\Pi 1} + n$
$iv_{LQ} := iv_{VH} + nvmax$
if *setLf*
    $iv_{\Psi 1} := iv_{LQ} + nvmax \cdot nf$
else
    $iv_{\Psi 1} := iv_{LQ}$
$bound := iv_{\Psi 1} + n$
Assert($bound - 1 \leq liv$,102)
$iv_{V1} := 50$
$iv_{VV1} := iv_{V1} + nvmax \cdot d$

$iv_{\Xi 1} := iv_{VV1} + (d+1) \cdot nvmax$
$iv_{WORK1} := iv_{\Xi 1} + ncmax$
$iv_{WORK2} := iv_{WORK1} + n$
$iv_{WORK3} := iv_{WORK2} + nf$
$iv_{VV2} := iv_{WORK3} + iv_{DIM} \cdot nf$
$iv_{LF} := iv_{VV2} + (d+1) \cdot nvmax$
if *setLf*
    $iv_{WORK4} := iv_{LF} + (d+1) \cdot nvmax \cdot nf$
else
    $iv_{WORK4} := iv_{LF}$
$bound := iv_{WORK4} + nf$
Assert$(bound - 1 \leq lv, 103)$
$v_F := f$
$v_{FCELL} := 0.05$
$v_{FDIAM} := 0.$
$v_{RCOND} := 1.$

$bound, d \in \mathbf{Z}$
$f \in \mathbf{R}$
$i \in \mathbf{Z}$
$ideg \in \mathbf{Z}, In$
$iv \in \mathbf{Z}^{liv}, Old$
$j, liv, lv, n \in \mathbf{Z}$
$ncmax \in \mathbf{Z}$, ... maximum number of cells
$nf \in \mathbf{Z}$
$nvmax \in \mathbf{Z}$, ... maximum number of vertices
$setLf$: *Logical*, *In*
$v \in \mathbf{R}^{lv}, Old$
$vc, version \in \mathbf{Z}$
Include: iv routines
 /fs/pyxis6/ehg/a/loess/lowesd.g
Wed May 29 14:00:17 1991

s := lowese(iv,liv,lv,wv,m,z)

Assert($iv_{SEQ} \neq 172,172$)
Assert($iv_{SEQ} = 173,173$)
ehg133(iv_N,iv_D,iv_{VC},iv_{NVMAX},iv_{NC},iv_{NCMAX},$iv_{iv_{A1}}$,$iv_{iv_{C1}}$,$iv_{iv_{HI1}}$,$iv_{iv_{LO1}}$,
$wv_{iv_{V1}}$,$wv_{iv_{VV1}}$,$wv_{iv_{E1}}$,m,z,s
)

$iv \in \mathbf{Z}^{\cdot}$,*Old*
$m \in \mathbf{Z}$
$s \in \mathbf{R}^{m}$,*Old*
$wv \in \mathbf{R}^{\cdot}$,*Old*
$z \in \mathbf{R}^{m \times 1}$,*Old*
Include: kd routines iv approx lin
/fs/pyxis6/ehg/a/loess/lowese.g
Wed May 29 14:01:53 1991

s := lowesf(xx,yy,ww,iv,liv,lv,wv,m,z,L,
ihat)

Assert($171 \le iv_{SEQ}$ **and** $iv_{SEQ} \le 174$,171)
iv_{SEQ} := 172
Assert($iv_{NVMAX} \ge iv_{NF}$,186)
s := l2fit(z,m,m,iv_N,iv_D,iv_{NF},wv_F,xx,$iv_{iv_{\Pi 1}}$,yy,
ww,iv_{KERNEL},iv_{DIM},$wv_{iv_{WORK1}}$,$wv_{iv_{WORK2}}$,$wv_{iv_{WORK3}}$,0,L,*ihat*,$wv_{iv_{WORK4}}$,
wv_{RCOND},iv_{SING},iv_{NDIST},iv_{DEG},$iv_{CDEG:CDEG+8}$
)

$L \in \mathbf{R}^{m \times n}$,*Old*($m$,·)
ihat $\in \mathbf{Z}$
$iv \in \mathbf{Z}^{\cdot}$,*Old*
$m \in \mathbf{Z}$
$s \in \mathbf{R}^{m}$,*Old*
$wv \in \mathbf{R}^{\cdot}$,*Old*
$ww \in \mathbf{R}^{\cdot}$,*Old*
$xx \in \mathbf{R}^{\cdot}$,*Old*,*Matrix*
$yy \in \mathbf{R}^{\cdot}$,*Old*
$z \in \mathbf{R}^{m \times 1}$,*Old*
Include: kd routines iv approx lin
/fs/pyxis6/ehg/a/loess/lowesf.g
Mon May 27 22:15:07 1991

lowesl(iv,liv,lv,wv,m,z,L)

Assert($iv_{SEQ} \neq 172,172$)
Assert($iv_{SEQ} = 173,173$)
Assert($iv_{WORK4} \neq iv_{LF},175$)
ehg191($m,z,L,iv_D,iv_N,iv_{NF},iv_{NV},iv_{NCMAX},iv_{VC},iv_{iv_{A1}}$,
$wv_{iv_{S1}},iv_{iv_{LO1}},iv_{iv_{HI1}},iv_{iv_{C1}},wv_{iv_{V1}},iv_{NVMAX},wv_{iv_{VV2}},wv_{iv_{LF}},iv_{iv_{LQ}}$
)

$L \in \mathbf{R}^{m \times n}, Old(m,\cdot), Out$
$iv \in \mathbf{Z}^{\cdot}, Old$
$m \in \mathbf{Z}, In$
$wv \in \mathbf{R}^{\cdot}, Old$
$z \in \mathbf{R}^{m \times 1}, Old, In$
Include: kd routines iv
/fs/pyxis6/ehg/a/loess/lowesl.g
Mon Jun 3 14:04:25 1991

lowesp($n,y,\hat{y},pwgts,rwgts,\pi,\tilde{y}$)

... median absolute deviation

$\tilde{y} := |y-\hat{y}|\cdot\sqrt{pwgts}$

$\pi := Identity$

$m := \text{Floor}(n/2)+1$

ehg106$(1,n,m,1,\tilde{y},\pi,n)$

if $m > n-m+1$

ehg106$(1,m-1,m-1,1,\tilde{y},\pi,n)$

$mad := \dfrac{\tilde{y}_{\pi_{m-1}}+\tilde{y}_{\pi_m}}{2}$

else

$mad := \tilde{y}_{\pi_m}$

... magic constant

$c := \dfrac{(6\cdot mad)^2}{5}$

$\tilde{y} := 1-\dfrac{(y-\hat{y})^2\cdot pwgts}{c}$

$\tilde{y} *= \sqrt{rwgts}$

$c := n/\text{Sum}(\tilde{y})$

... pseudovalues

$\tilde{y} := \hat{y}+(c\cdot rwgts)\cdot(y-\hat{y})$

$c \in \mathbf{R}$

$m \in \mathbf{Z}$

$mad \in \mathbf{R}$

$n \in \mathbf{Z}, In$

$\pi \in \mathbf{Z}^n, Permutation, Old$

$pwgts \in \mathbf{R}^n, Old, In$

$rwgts \in \mathbf{R}^n, Old, In$

$y \in \mathbf{R}^n, Old, In$

$\hat{y} \in \mathbf{R}^n, Old, In$

$\tilde{y} \in \mathbf{R}^n, Old, Out$

Include: lin

/fs/pyxis6/ehg/a/loess/lowesp.g

Tue May 29 15:33:26 1990

lowesr(y,iv,liv,lv,wv)

Assert($iv_{SEQ} \neq 172$,172)
Assert($iv_{SEQ} = 173$,173)
ehg192(y,iv_D,iv_N,iv_{NF},iv_{NV},iv_{NVMAX},$wv_{iv_{W1}}$,$wv_{iv_{LF}}$,$iv_{iv_{LQ}}$)

$iv \in \mathbf{Z}^{\cdot}$,*Old*,*In*
$wv \in \mathbf{R}^{\cdot}$,*Old*,*In*
Include: kd iv
/u3/ehg/a/loess/lowesr.g
Tue Apr 3 21:31:13 1990

lowesw(res, n, rw, π)

... tranliterated from Devlin's ratfor
... find median of absolute residuals
    $rw := |res|$
    $\pi := Identity$
    $nh := \text{Floor}(n/2)+1$
... partial sort to find 6*mad
    ehg106$(1, n, nh, 1, rw, \pi, n)$
    if $nh > n - nh + 1$
        ehg106$(1, nh-1, nh-1, 1, rw, \pi, n)$
        $cmad := 3 \cdot (rw_{\pi_{nh}} + rw_{\pi_{nh-1}})$
    else
        $cmad := 6 \cdot rw_{\pi_{nh}}$
$rsmall := \text{r1mach}(1)$
if $cmad < rsmall$
    $rw := 1$
else
    for $1 \le i \le n$
        if $rw_i > cmad \cdot 0.999$
            $rw_i := 0$
        else if $rw_i > cmad \cdot 0.001$
            $rw_i := (1-(rw_i/cmad)^2)^2$
        else
            $rw_i := 1$

$cmad \in \mathbf{R}$
$i, n, nh \in \mathbf{Z}$
$\pi \in \mathbf{Z}^n, Permutation, Old$
$res \in \mathbf{R}^n, Old$
$rsmall \in \mathbf{R}$
$rw \in \mathbf{R}^n, Old$
Include: lin
 /fs/pyxis6/ehg/a/loess/lowesw.g
Wed May 29 14:24:29 1991

ehg124

rbuild($ll, uu, d, n, n_{vert}, n_{cell}, ncmax, vc, x, \pi,$
$a, \xi, lo, hi, c, v, vhit, nvmax, f_{cell}, f_{diam},$
dd)

$p := 1$
$l := ll$
$u := uu$
$lo_p := l$
$hi_p := u$
while $p \le n_{cell}$
diag $:= v_{c_{vc,p},1:dd} - v_{c_{1,p},1:dd}$
$diam :=$ || diag ||$_2$
$leaf :=$ ($u - l + 1 \le f_{cell}$ **or** $diam \le f_{diam}$) **or** ($n_{cell} + 2 > ncmax$ **or** $n_{vert} + vc/2 > nvmax$)
if $!leaf$
$\sigma :=$ spread(l, u, dd, x, π, n)
$k :=$ iMax(σ)
$m :=$ Floor($\frac{l+u}{2}$)
PartialSort($l, u, m, x_{:,k}, \pi$)
... all ties go with hi son
while $m > 1$ **and** $x_{\pi_{m-1},k} = x_{\pi_m,k}$
m −= 1
$leaf := v_{c_{1,p},k} = x_{\pi_m,k}$ **or** $v_{c_{vc,p},k} = x_{\pi_m,k}$
if $leaf$
$a_p := 0$
else
$a_p := k$
$\xi_p := x_{\pi_m,k}$
... left son
n_{cell} += 1
$lo_p := n_{cell}$
$lo_{n_{cell}} := l$
$hi_{n_{cell}} := m$
... right son
n_{cell} += 1
$hi_p := n_{cell}$
$lo_{n_{cell}} := m+1$
$hi_{n_{cell}} := u$
cpvert($p, n_{vert}, v, vhit, nvmax, d, k, \xi_p, 2^{k-1}, 2^{d-k},$
$c_{:,p}, c_{:,lo_p}, c_{:,hi_p}$
)

p += 1
$l := lo_p$
$u := hi_p$

---

cpvert≡ehg125
diag∈ $\mathbf{R}^{dd}$,*Old*(*dMAX*)
diam∈ **R**
k∈ **Z**
l∈ **N**
leaf: *Logical*
ll∈ **Z**
m∈ **N**
p∈ **Z**, ... current cell; root for points l to u
σ∈ $\mathbf{R}^{dd}$,*Old*(*dMAX*)
spread≡ehg129
u∈ **N**
uu∈ **Z**
Include: kd routines approx lin
/u3/ehg/a/loess/rbuild.g
Thu Jan 25 16:36:25 1990

ehg129

σ := spread(l,u,d,x,π,n)

for $1 \le k \le d$
  $\alpha := \infty$
  $\beta := -\infty$
  for $l \le i \le u$
    $t := x_{\pi_i,k}$
    $\alpha := \text{Min}(\alpha, x_{\pi_i,k})$
    $\beta := \text{Max}(\beta, t)$
  $\sigma_k := \beta - \alpha$

$\alpha,\beta \in \mathbf{R}$
$i,k,l \in \mathbf{Z}$
$\sigma \in \mathbf{R}^d, Old$
$t \in \mathbf{R}$
$u \in \mathbf{Z}$
Include: kd lin
/u3/ehg/a/loess/spread.g
Mon Mar 23 12:29:45 1987

ehg137

vleaf($z,\kappa,leaf,nleaf,d,n_{vert},nvmax,ncmax,vc,a,$
ξ,lo,hi,c,v)

... find leaf cells affected by z
$stacktop := 0$
$p := 1$
$nleaf := 0$
while $p>0$
  if $a_p = 0$ ... leaf
    $nleaf$ += 1
    $leaf_{nleaf} := p$
    $p := Pop$
  else if $z_{a_p} = \xi_p$
    Push(hi_p)
    $p := lo_p$
  else if $z_{a_p} < \xi_p$
    $p := lo_p$
  else
    $p := hi_p$
Assert($nleaf \le vcMAX$,185)

$STACKMAX \rightarrow 20$
$Push(\%?,\%mp) \rightarrow \{stacktop$ += $1,Assert(stacktop \le STACKMAX,187),pstack_{stacktop} := \%mp\}$
$\%? := \%mp := Pop \rightarrow \{$ *bold* if
$(stacktop \ge 1,\%mp := pstack_{stacktop},$**else**$,\%mp := 0),stacktop := Max(0,stacktop-1)\}$
$leaf \in \mathbf{Z}^{vcMAX},Old,$ ... indices of leaf cells
$nleaf \in \mathbf{Z},$ ... number of leaf cells found
$p \in \mathbf{Z},$ ... cell under consideration
$stacktop \in \mathbf{Z},$ ... index of top element in stack
$z \in \mathbf{R}^d,Old,$ ... vertex
Include: kd routines
/u3/ehg/a/loess/vleaf.g
Mon Dec 7 09:30:35 1987